Estimating Dynamic Treatment Effects in Event Studies with Heterogeneous Treatment Effects

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Introduction

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- Researchers often estimate dynamic treatment effects by the estimates for coefficients μ_ℓ in a (dynamic) two-way FE specification that resembles the following:

$$Y_{i,t} = \alpha_i + \lambda_t + \sum_{\ell} \mu_\ell D_{i,t}^{\ell} + \upsilon_{i,t}$$

where $D_{i,t}^{\ell}$ is an indicator for ℓ periods relative to i's initial treatment ($\ell = 0$ is the period of initial treatment).

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  where $D_{i,t}^{\ell}$ is an indicator for ℓ periods relative to i's initial treatment ($\ell = 0$ is the period of initial treatment).
- Goal of our project: characterize μ_ℓ under heterogenous treatment effect in **event studies**
  - absorbing treatment
  - variation in treatment timing

Examples of event studies

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- Dobkin et al. (2018): the dynamic effects of a hospitalization, where the initial treatment is the initial hospital admission

Overview

- Intuition: compare earlier cohort with later cohort to estimate the dynamic effect
- Report estimates for relative period coefficients μ_ℓ as estimates for dynamic effects
- Potentially problematic when there are multiple cohorts:
  - Decompose μ_ℓ in terms of cohort-specific effects
  - Demonstrate potential contamination from $\ell' \neq \ell$ due to heterogeneity

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    - Demonstrate potential contamination from $\ell' \neq \ell$ due to heterogeneity
- Event studies can show up under different names
    - "staggered adoption" (Athey and Imbens, 2018)
    - "stepped wedge design" (Ellenberg, JAMA 2018)

For today's talk

- Literature review
- Cast event studies in a potential outcomes framework
- Decomposition
- Alternative methods
- Empirical illustration

Literature review

Literature review: 1/2

- Active literature on the causal interpretations of two-way fixed effects regressions (Athey and Imbens, 2018; Borusyak and Jaravel, 2017; Callaway and Sant'Anna, 2020; de Chaisemartin and D'Haultfœuille, 2020; Goodman-Bacon, 2018)
- Mostly focus on "static" specifications:

$$Y_{i,t} = \alpha_i + \lambda_t + \mu D_{i,t} + \upsilon_{i,t}$$

- Decompose μ into non-convex combination of heterogeneous treatment effects
- We focus on "dynamic" specifications

Literature review: 2/2

- de Chaisemartin and D'Haultfœuille (2020) propose diagnostic tools and alternative estimators
- Callaway and Sant'Anna (2020) allows for conditioning on time-varying covariates
- We propose simple regression-based alternative estimation strategy for dynamic treatment effects

Potential outcome framework

Potential outcome framework: 1/3

- A random sample of N units observed over $T+1$ time periods with i.i.d. observations $\{Y_{i,t}, D_{i,t}\}_{t=0}^{T}$
- Treatment status $D_{i,t} \in \{0, 1\}$
- Treatment is actually a sequence $\{D_{i,s}\}_{s=0}^{T}$.
    - In event studies, can be identified with the scalar $E_i = \min\{t : D_{i,t} = 1\}$, the period of initial treatment
    - Let $E_i = \infty$ for those never treated
    - **Treatment cohort**: a group $\{i : E_i = e\}$ of unit first treated at the same time
- Results hold with or without a never treated cohort (control cohort).

Potential outcome framework: 2/3

- Potential outcome $Y_{i,t}^{e}$ is the outcome in response to treatment that first starts in period e
- "Baseline outcome" $Y_{i,t}^{\infty}$ is the potential outcome if never treated
- Observed outcome is therefore

$$Y_{i,t} = Y_{i,t}^{E_i} = Y_{i,t}^{\infty} + \sum_{0 \leq e \leq T} \left(Y_{i,t}^{e} - Y_{i,t}^{\infty} \right) \cdot \mathbf{1}\left\{ E_i = e \right\}$$

- Unit-level treatment effect $Y_{i,t} - Y_{i,t}^{\infty}$

Potential outcome framework: 3/3

- "Building blocks" for causal interpretation are cohort-specific average treatment effect on the treated (CATT) ℓ periods from initial treatment:

$$CATT_{e,\ell} = E[Y_{i,e+\ell} - Y^{\infty}_{i,e+\ell} \mid E_i = e]$$

- This object coincides with the "group-time average treatment effect" studied by Callaway and Sant'Anna (2020)
- Next use potential outcome notations to articulate identifying assumptions underlying the dynamic specification

Identifying assumptions: 1/3

Assumption 1.

(Parallel trends in baseline outcome.) For all $s \neq t$*, the* $E[Y^{\infty}_{i,t} - Y^{\infty}_{i,s} | E_i = e]$ *is the same for all* $e \in supp(E_i)$*.*

- Potential violation: Ashenfelter's dip

Identifying assumptions: 2/3

Assumption 2.
(No anticipation.) There is no treatment effect in pre-treatment periods i.e. $E[Y^{e}_{i,e+\ell} - Y^{\infty}_{i,e+\ell} \mid E_i = e] = 0$ *for all* $e \in supp(E_i)$ *and all* $\ell < 0$.

- Potential violation: Hendren (2017) shows that knowledge of future job loss leads to decreases in consumption due to anticipation
- Similar to Malani and Reif (2015) and Botosaru and Gutierrez (2018)

Identifying assumptions: 3/3

Assumption 3.

(Treatment effect homogeneity.) For each relative period ℓ*,* $CATT_{e,\ell}$ *does not depend on cohort* e *and is equal to* ATT_ℓ*.*

Potential violations:

- Effects vary with covariates
- Selection into treatment timing based on effects
- Calendar time-varying effects (e.g. macroeconomic conditions could govern the effects on labor market outcomes)

Decompose the dynamic specification

Dynamic specification

$$Y_{i,t} = \alpha_i + \lambda_t + \sum_{\ell=-K}^{-2} \mu_\ell D_{i,t}^\ell + \sum_{\ell=0}^{L} \mu_\ell D_{i,t}^\ell + \upsilon_{i,t}$$

- μ_ℓ denotes the population regression coefficient, i.e. the probability limit of the associated OLS estimator $\widehat{\mu}_\ell$
  - Included relative periods collected in $g^{incl} = \{-K, \ldots, 0, \ldots, L\}$
  - Excluded relative periods collected in $g^{excl} = \{-T, \ldots, -K-1, -1, L+1, \ldots, T\}$
  - Excluding some relative periods can be necessary due to multi-collinearity (Borusyak and Jaravel, 2017)

Properties of TWFE weights

Proposition 1.

Under parallel trends, we can write μ_ℓ as a linear combination of $CATT_{e,\ell}$ as well as $CATT_{e,\ell'}$ from other relative periods $\ell' \neq \ell$,

$$\mu_\ell = \sum_e \omega^\ell_{e,\ell} CATT_{e,\ell} + \sum_{\ell' \neq \ell, \ell' \in g^{incl}} \sum_e \omega^\ell_{e,\ell'} CATT_{e,\ell'} + \sum_{\ell' \in g^{excl}} \sum_e \omega^\ell_{e,\ell'} CATT_{e,\ell'}$$

1. For own relative period: weights sum to one
2. For other relative periods included in the specification: weights sum to zero for each $\ell' \neq \ell$
3. For relative periods excluded from the specification: $\sum_{\ell' \in g^{excl}} \sum_e \omega^\ell_{e,\ell'} = -1$

If the weights $\omega_{e,\ell'}$ are non-zero, then effects from $\ell' \neq \ell$ can potentially contaminate the interpretation of μ_ℓ

Deriving the weights: 1/2

Consider a saturated regression:

$$\begin{aligned} Y_{i,t} = & \sum_e \alpha_e \cdot \mathbf{1}\{E_i = e\} + \sum_s \lambda_s \cdot \mathbf{1}\{t = s\} \\ & + \sum_{\ell \in g^{incl}} \sum_e \gamma_{e,\ell} \cdot \left(D_{i,t}^{\ell} \cdot \mathbf{1}\left\{E_i = e\right\} \right) \\ & + \sum_{\ell' \in g^{excl}} \sum_e \gamma_{e,\ell'} \cdot \left(D_{i,t}^{\ell'} \cdot \mathbf{1}\left\{E_i = e\right\} \right) + \epsilon_{i,t} \end{aligned}$$

Deriving the weights: 1/2

Consider a saturated regression:

$$
\begin{aligned}
Y_{i,t} = & \sum_e \alpha_e \cdot \mathbf{1}\{E_i = e\} + \sum_s \lambda_s \cdot \mathbf{1}\{t = s\} \\
& + \sum_{\ell \in g^{incl}} \sum_e \gamma_{e,\ell} \cdot \left(D^{\ell}_{i,t} \cdot \mathbf{1}\left\{E_i = e\right\} \right) \\
& + \sum_{\ell' \in g^{excl}} \sum_e \gamma_{e,\ell'} \cdot \left(D^{\ell'}_{i,t} \cdot \mathbf{1}\left\{E_i = e\right\} \right) + \epsilon_{i,t}
\end{aligned}
$$

Under parallel trends, we know the population coefficients:

$$
\begin{aligned}
Y_{i,t} = & \sum_e E[Y^{\infty}_{i,0} \mid E_i = e] \cdot \mathbf{1}\{E_i = e\} + \sum_s E[Y^{\infty}_{i,s} - Y^{\infty}_{i,0}] \cdot \mathbf{1}\{t = s\} \\
& + \sum_{\ell \in g^{incl}} \sum_e CATT_{e,\ell} \cdot \left(D^{\ell}_{i,t} \cdot \mathbf{1}\left\{E_i = e\right\} \right) \qquad (1) \\
& + \sum_{\ell' \in g^{excl}} \sum_e CATT_{e,\ell'} \cdot \left(D^{\ell'}_{i,t} \cdot \mathbf{1}\left\{E_i = e\right\} \right) + \epsilon_{i,t} \qquad (2)
\end{aligned}
$$

Deriving the weights: 2/2

Apply the OVB formula to derive the expression for μ_ℓ in terms of $CATT_{e,\ell'}$:

- finds its associated regressor in the saturated regression $D_{i,t}^{\ell'} \cdot \mathbf{1}\left\{E_i = e\right\}$
- multiplies it with the regression coefficients from

$$D_{i,t}^{\ell'} \cdot \mathbf{1}\left\{E_i = e\right\} = \alpha_i + \lambda_t + \sum_{\ell \in g^{incl}} \omega_{e,\ell'}^{\ell} D_{i,t}^{\ell} + u_{i,t}$$

Intuition for the weights

$$D_{i,t}^{\ell'} \cdot \mathbf{1}\{E_i = e\} = \alpha_i + \lambda_t + \sum_{\ell \in g^{incl}} \omega_{e,\ell'}^{\ell} D_{i,t}^{\ell} + u_{i,t}$$

- We provide code for calculating these weights `eventstudyweights`
- $\omega_{e,\ell'}^{\ell} \neq 0$ even for $\ell' \neq \ell$ because the panel cannot be balanced in both calendar and relative times when there are multiple cohorts
- Magnitude of $\omega_{e,\ell'}^{\ell}$ determines how sensitive μ_ℓ is to $CATT_{e,\ell'}$
- Can invalidate a test for pre-trend

Invalidity of the pre-trend test

Proposition 2.

Under parallel trends and no anticipation, we can write a lead coefficient μ_ℓ for $\ell < 0$ as a linear combination of post-treatment $CATT_{e,\ell'}$ for all $\ell' \geq 0$:

$$\mu_\ell = \sum_{\ell' \geq 0} \sum_{e} \omega^{\ell}_{e,\ell'} CATT_{e,\ell'} + \sum_{\ell' \in g^{excl}, \ell' \geq 0} \sum_{e} \omega^{\ell}_{e,\ell'} CATT_{e,\ell'}$$

- Even if $CATT_{e,\ell} = 0$ for all $\ell < 0$, can still get non-zero μ_ℓ due to contamination from post-treatment periods

If willing to impose treatment effect homogeneity...

Proposition 3.

Under parallel trends and treatment effect homogeneity, we have $CATT_{e,\ell} = ATT_\ell$ *for a given* ℓ *and*

$$\mu_\ell = ATT_\ell + \sum_{\ell' \in g^{excl}} \omega_{\ell'}^{\ell} ATT_{\ell'}.$$

- Additional term drops out when excluded periods have zero effect, otherwise can be thought of as a type of "normalization": $\sum_{\ell' \in g^{excl}} \omega_{\ell'}^{\ell} = -1$

Alternative methods

Alternative methods for estimating dynamic treatment effects

Consider the estimand:

$$\nu_\ell = \sum_e CATT_{e,\ell} Pr\{E_i = e \mid E_i \in [-\ell, T-\ell]\}$$

We propose the interaction-weighted (IW) estimator à la Gibbons et al. (2018)

1. Estimate $CATT_{e,\ell}$ by

$$Y_{i,t} = \alpha_i + \lambda_t + \sum_{e \notin C} \sum_{\ell \neq -1} \delta_{e,\ell} (\mathbf{1}\{E_i = e\} \cdot D_{i,t}^{\ell}) + \epsilon_{i,t}$$

2. Estimate the weights by sample shares
3. Form the IW estimator $\widehat{\nu}_\ell$ by

$$\widehat{\nu}_\ell = \sum_e \widehat{\delta}_{e,\ell} \widehat{Pr}\{E_i = e \mid E_i \in [-\ell, T-\ell]\}$$

Validity of the IW estimator

Proposition 4.

Under parallel trends, no anticipation and some regularity conditions, the IW estimator $\widehat{\nu}_\ell$ is consistent and asymptotically normal:

$$\sqrt{N}\left(\widehat{\nu}_\ell - \nu_\ell\right) \to_d N\left(0, \Sigma\right)$$

- Note that $\widehat{\delta}_{e,\ell}$ is a difference-in-differences estimator for $CATT_{e,\ell}$
- Σ accounts for asymptotic variance from both $\widehat{\delta}_{e,\ell}$ and the sample shares

Empirical illustration

Consequences of hospitalization (Dobkin et al., 2018)

Dynamic two-way fixed effects regression

$$
\begin{aligned}
Y_{i,t} = & \alpha_i + \lambda_t + \mu_{-3} D_{i,t}^{-3} + \mu_{-2} D_{i,t}^{-2} \\
& + \mu_0 D_{i,t}^{0} + \mu_1 D_{i,t}^{1} + \mu_2 D_{i,t}^{2} + \mu_3 D_{i,t}^{3} + \upsilon_{i,t}
\end{aligned}
$$

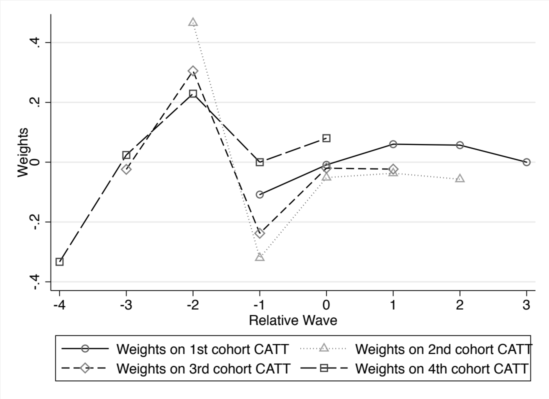
- $Y_{i,t}$ out-of-pocket medical spending; $D_{i,t}^{\ell}$ period relative to initial hospitalization
- Balanced panel of $N = 656$ over $t \in \{0, \dots, 4\}$ from Health and Retirement Study (HRS)
- Four cohorts $E_i \in \{1, 2, 3, 4\}$ with $\ell \in \{-3, -2, 0, 1, 2, 3\}$ included but $\ell \in \{-4, -1\}$ excluded

Decomposition of μ_{-2}

$$\mu_{-2} = \underbrace{\sum_{e=1}^{4} \omega_{e,-2}^{-2} CATT_{e,-2}}_{\text{own period}}$$

$$+ \sum_{\ell \in \{-3,0,1,2,3\}} \underbrace{\sum_{e=1}^{4} \omega_{e,\ell}^{-2} CATT_{e,\ell}}_{\text{other included period}}$$

$$+ \sum_{\ell' \in \{-4,-1\}} \underbrace{\sum_{e=1}^{4} \omega_{e,\ell'}^{-2} CATT_{e,\ell'}}_{\text{excluded period}}$$



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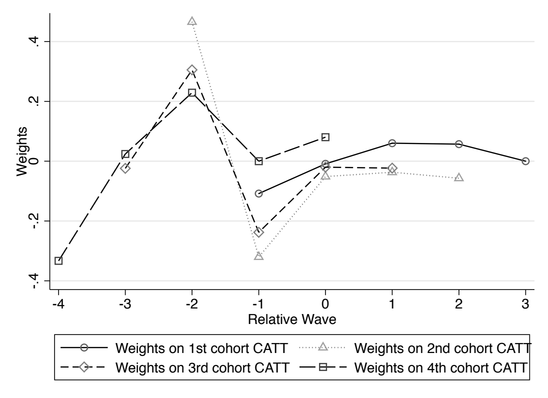

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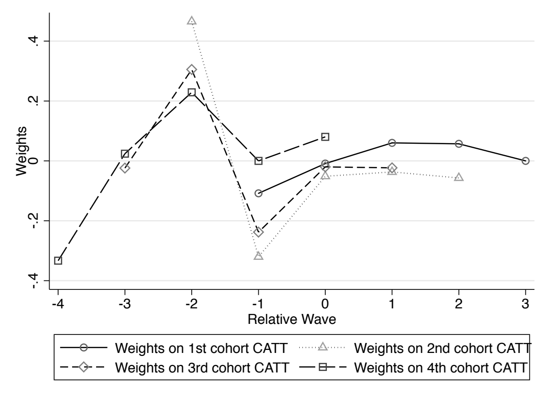

Effect of Hospitalization on Out-of-pocket Medical Spending

	FE	IW		$CATT_{e,\ell}$	
ℓ Relative to Hospitalization	$\widehat{\mu}_\ell$	$\widehat{\nu}_\ell$	$\widehat{\delta}_{1,\ell}$	$\widehat{\delta}_{2,\ell}$	$\widehat{\delta}_{3,\ell}$
-3	149	591	-	-	591
	(792)	(1273)			(1273)
-2	203	353	-	299	411
	(480)	(698)		(967)	(1030)
-1	0	0	0	0	0
0	3,013	2,960	2,826	3,031	3,092
	(511)	(543)	(1038)	(704)	(998)
1	888	530	825	107	-
	(664)	(587)	(912)	(653)	
2	1,172	800	800	-	-
	(983)	(1010)	(1010)		
3	1,914	-	-	-	-
	(1426)				

Conclusion

- Decompose the relative period coefficient μ_ℓ from dynamic specification for event studies
- Demonstrate that under treatment effects heterogeneity μ_ℓ may pick up spurious terms consisting of treatment effects from periods other than ℓ
- Propose "interaction-weighted" (IW) estimator that is more robust toward heterogeneity

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- Thank you!